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The Big Flow

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The late infall of cold dark matter onto an isolated galaxy, such as our own, produces streams and caustics in its halo. The outer caustics are topological spheres whereas the inner caustics are rings. The self-similar model of galactic halo formation predicts that the caustic ring radii a_n follow the approximate law $a_n \sim 1/n$. In a study of 32 extended and well-measured external galactic rotation curves evidence was found for this law. In the case of the Milky Way, the locations of eight sharp rises in the rotation curve fit the prediction of the self-similar model at the 3% level. Moreover, a triangular feature in the IRAS map of the galactic plane is consistent with the imprint of a ring caustic upon the baryonic matter. These observations imply that the dark matter in our neighborhood is dominated by a single flow. Estimates of that flow's density and velocity vector are given.

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1 Introduction

There are compelling reasons to believe that the dark matter of the universe is constituted in large part of non-baryonic collisionless particles with very small primordial velocity dispersion, such as axions and/or weakly interacting massive particles (WIMPs) [1]. Generically, such particles are called cold dark matter (CDM). Knowledge of the distribution of CDM in galactic halos, and in our own halo in particular, is of paramount importance to understanding galactic structure and predicting signals in experimental searches for dark matter.

One should expect this dark matter to form caustics. A caustic is a place in physical space where the density is very large because the sheet on which the dark matter particles lie in phase-space has a fold there. Caustics are commonplace in the propagation of light. An instructive example is given by the sharp luminous lines at the bottom of a swimming pool on a breezy sunny day. Two conditions must be satisfied for caustics to occur generically. First, the propagation must be collisionless. Second the flow must have low velocity dispersion. Light propagation is collisionless, and the flow of light from a point source has zero velocity dispersion. Thus caustics are common in light. Caustics in ordinary matter are very unusual because ordinary matter is not normally collisionless. But CDM is collisionless and has very small velocity dispersion. This guarantees that caustics are common in the distribution of CDM.

The primordial velocity dispersion of the cold dark matter candidates is indeed very small, of order

$$\delta v_a(t) \sim 3 \cdot 10^{-17} \left(\frac{10^{-5} eV}{m_a}\right) \left(\frac{t_0}{t}\right)^{2/3} \tag{1}$$

for axions, and

$$\delta v_W(t) \sim 10^{-11} \left(\frac{GeV}{m_W}\right)^{1/2} \left(\frac{t_0}{t}\right)^{2/3} \tag{2}$$

for WIMPs. Here t_0 is the present age of the universe and m_a and m_W are respectively the masses of the axion and WIMP. The small velocity dispersion means that the dark matter particles lie on a thin 3-dim. sheet in 6-dim. phase-space. The thickness of the sheet is δv. The sheet cannot break and hence its evolution is constrained by topology.

Where a galaxy forms, the sheet wraps up in phase-space, turning clockwise in any two dimensional cut $(x, \dot{x})$ of that space. x is the physical space coordinate in an arbitrary direction and $\dot{x}$ its associated velocity. The outcome of this process is a discrete set of flows at any physical point in a galactic halo [2]. Two flows are associated with particles falling through the galaxy for the first time ($n = 1$), two other flows are associated with particles falling through the galaxy for the second time ($n = 2$), and so on. Scattering in the gravitational wells of inhomogeneities in the galaxy (e.g. molecular clouds and globular clusters) are ineffective in thermalizing the flows with low values of n.

Caustics appear wherever the projection of the phase-space sheet onto physical space has a fold [3, 4, 5, 6]. Generically, caustics are surfaces in physical space. On one side of the caustic surface there are two more flows than on the other. At the surface, the dark matter density is very large. It diverges there in the limit of zero velocity dispersion. There are two types of caustics in the halos of galaxies, inner and outer. The outer caustics are topological spheres surrounding the galaxy. They are located near where a given outflow reaches its furthest distance from the galactic center before falling back in. The inner caustics are rings [3]. They are located near where the particles with the most angular momentum in a given inflow reach their distance of closest approach to the galactic center before going back out. A caustic ring is a closed tube whose cross-section is a D_{-4} (also called *elliptic umbilic*) catastrophe [6]. The existence of these caustics and their topological properties are independent of any assumptions of symmetry.

As was mentioned earlier, the primordial velocity dispersion of the leading cold dark matter candidates is extremely small. However, to a coarse-grained observer, the dark matter falling onto a galaxy may have additional velocity dispersion because the phase-space sheet on which the dark matter particles lie may be wrapped up on scales which are small compared to the galaxy as a whole. This effective velocity dispersion is associated with the clumpiness of the dark matter before it falls onto the galaxy. For the caustics in a galaxy not to be washed out, the effective velocity dispersion of the infalling dark matter must be much less than the rotation velocity of the galaxy, say less than 30 km/s for our galaxy. However, an upper bound of order 50 m/s can be obtained from observation, as explained below.

Primordial peculiar velocities are expected to be the same for baryonic and dark matter particles because they are caused by gravitational forces. Later the velocities of baryons and CDM differ because baryons collide with each other whereas CDM is collisionless. However, because angular momentum is conserved, the net angular momenta of the dark matter and baryonic components of a galaxy are aligned. Since the caustic rings are located near where the particles with the most angular momentum in a given infall are at their closest approach to the galactic center, they lie close to the galactic plane.

2 Caustic ring radii

A specific proposal has been made for the radii a_n of caustic rings [3]:

$$\{a_n : n = 1, 2, ...\} \simeq (39,\ 19.5,\ 13,\ 10,\ 8, ...)\text{kpc} \times \left(\frac{j_{\rm max}}{0.25}\right)\left(\frac{0.7}{h}\right)\left(\frac{v_{\rm rot}}{220\frac{\rm km}{\rm s}}\right) \qquad (3)$$

where h is the present Hubble constant in units of $100\,\text{km/(s Mpc)}$, $v_{\rm rot}$ is the rotation velocity of the galaxy and $j_{\rm max}$ is a parameter with a specific value for each halo. For large n, $a_n \sim 1/n$. Eq. 3 is predicted by the self-similar infall model [7, 8] of galactic halo formation. $j_{\rm max}$ is then the maximum of the dimensionless angular momentum j-distribution [8]. The self-similar model depends upon a parameter ϵ [7]. In CDM theories of large scale structure formation, ϵ is expected to be in the range 0.2 to 0.35 [8]. Eq. 3 is for $\epsilon = 0.3$. However, in the range $0.2 < \epsilon < 0.35$, the ratios a_n/a_1 are almost independent of ϵ. When $j_{\max}$ values are quoted below, $\epsilon = 0.3$ and $h = 0.7$ will be assumed.

Since the caustic rings lie close to the galactic plane, they cause bumps in the rotation curve, at the locations of the rings. In ref. [9] a set of 32 extended well-measured rotation curves was analyzed and statistical evidence was found for bumps distributed according to Eq. 3. That study suggests that the $j_{\max}$ distribution is peaked near 0.27. The rotation curve of NGC3198, one of the best measured, by itself shows three faint bumps which are consistent with Eq. 3 and $j_{\max} = 0.28$.

A recent paper [10] gives evidence for ring caustics in our own galaxy.

3 Ring caustics in the Milky Way

A detailed north inner galactic rotation curve was obtained [11] from the Massachusetts-Stony Brook Galactic Plane CO survey [12]. It exhibits a series of eight sharp rises in the range of (galactocentric) radii 3 to 7 kpc. For each, Table I lists the radius r_1 where the rise starts, the radius r_2 where it ends, and the increase Δv in rotation velocity. The rises are interpreted here as due to the presence of caustic rings of dark matter in the galactic plane. Each r_1 should therefore be identified with a caustic ring radius a_n, and $r_2 - r_1$ with the caustic ring width p_n [6]. The ring widths depend in a complicated way on the velocity distribution of the infalling dark matter at last turnaround [6] and are not predicted by the model. They also need not be constant along the ring. In Table I, the numbers in parentheses are for two less distinct rises between 7 kpc and our own radius $r_\odot$, taken to be 8.5 kpc.

The fourth column shows the caustic ring radii a_n^{I} of the $\epsilon = 0.3$ self-similar infall model fitted to the eight rises between 3 and 7 kpc, assuming that these are due to caustic rings $n = 7...14$ (fit I). This is a one-parameter ($j_{\max}$) fit minimizing $rmsd \equiv [\frac{1}{8}\sum_{n=7}^{14}(1 - \frac{a_n}{r_{1n}})^2]^{\frac{1}{2}}$. It yields $j_{\max} = 0.263$ and $rmsd = 3.1\%$. The fifth column shows the radii a_n^{II} assuming that the eight rises between 3 and 7 kpc are due to caustic rings $n = 6...13$ (fit II). In this case $j_{\max} = 0.239$ and $rmsd = 2.8\%$. Fits of similar quality are obtained for the other values of ϵ in the range 0.20 to 0.35, or by assuming simply $a_n \sim 1/n$. On the other hand, the assumption that the eight rises between 3 and 7 kpc are due to caustic rings $n = 6 + s...13 + s$, where s is an integer other than 0 or 1, yields considerably worse fits. Up to this point it is unclear whether $s = 0$ or 1 is preferred. However the two less distinct rises between 7 kpc and $r_\odot$ strongly suggest $s = 1$ since their r_1 values agree at the 2.6% level with ring radii a_5^{I} and a_6^{I}, but do not agree well with a_4^{II} and a_5^{II}. Henceforth $s = 1$ will be assumed.

The velocity increase due to a caustic ring is given by

$$\Delta v_n = v_{\mathrm{rot}} f_n \frac{\Delta I(\zeta_n)}{\cos\delta_n(0) + \phi_n'(0)\sin\delta_n(0)} \ . \tag{4}$$

The f_n, defined in ref. [3], are predicted by the self-similar infall model, but $\Delta I(\zeta_n), \delta_n(0)$ and $\phi'_n(0)$, defined in ref. [6], are not. Like the p_n, the latter parameters depend in a complicated way on the velocity distribution of the dark matter at last turnaround. On the basis of the discussion in ref. [6], the ratio on the RHS of Eq. 4 is expected to be of order one, but to vary from one caustic ring to the next. The size of these fluctuations is easily a factor two, up or down. The sixth column of Table I shows Δv_n with the fluctuating ratio set equal to one, i.e. $\bar{\Delta} v_n \equiv f_n v_{\rm rot}$.

For the reasons just stated, the fact that the observed Δv fluctuate by a factor of order 2 from one rise to the next is consistent with the interpretation that the rises are due to caustic rings. However the observed Δv (column 3) are typically a factor 5 larger than the velocity increases caused by the caustic rings acting alone (column 6). To account for the discrepancy I assume that the effect of the caustic rings is amplified by baryonic matter they have accreted. First I'll argue that the gas in the disk has sufficiently high density and low velocity dispersion for such an explanation to be plausible. Second I'll give observational evidence in support of the explanation.

The equilibrium distribution of gas is:

$$d_{\rm gas}(\vec{r}) = d_{\rm gas}(\vec{r}_0) \exp[-\frac{3}{< v_{\rm gas}^2 >}(\phi(\vec{r}) - \phi(\vec{r}_0))] \ , \tag{5}$$

where d is density and ϕ gravitational potential. In the solar neighborhood, $d_{\rm gas} \simeq 3 \cdot 10^{-24} \frac{\rm gr}{\rm cm^3}$ [13], which is comparable to the density of dark matter inside the tubes of caustic rings near us. From the scale height of the gas [13] and the assumption that it is in equilibrium with itself and the other disk components, I estimate $\langle v_{\rm gas}^2 \rangle^{\frac{1}{2}} \simeq 8$ km/s. The variation in the gravitational potential due to a caustic ring over the size of the tube is of order $\Delta\phi_{\rm CR} \simeq 2 f v_{\rm rot}^2 p/a \simeq (5 \frac{\rm km}{\rm s})^2$. Because $\frac{3}{<v_{\rm gas}^2>} \Delta\phi_{\rm CR}$ is of order one, the caustic rings have a large effect on the distribution of gas in the disk. The accreted gas amplifies and can dominate the effect of the caustic rings on the rotation curve. To check whether this hypothesis is consistent with the shape of the rises would require detailed modeling, as well as detailed knowledge on how the rotation curve is measured. In the meantime, I found observational evidence in its support.

The accreted gas may reveal the location of caustic rings in maps of the sky. Looking tangentially to a ring caustic from a vantage point in the plane of the ring, one may recognize the tricusp [6] shape of the D_{-4} catastrophe. The IRAS map of the galactic disk in the direction of galactic coordinates $(l, b) = (80^\circ, 0^\circ)$ shows a triangular shape which is strikingly reminiscent of the cross-section of a ring caustic. The vertices of the triangle are at $(83.5^\circ, 0.3^\circ), (77.3^\circ, 3.5^\circ)$ and $(77.4^\circ, -2.7^\circ)$ galactic coordinates. Images can be obtained from the Skyview Virtual Observatory (http://skyview.gsfc.nasa.gov/). The shape is correctly oriented with respect to the galactic plane and the galactic center. Moreover its position is consistent with that of a rise in the rotation curve, the one between 8.28 and 8.38 kpc ($n = 5$ in fit I). The caustic ring radius implied by the image is 8.31 kpc, and its dimensions are $p = 134$ pc and $q = 200$ pc, in the directions parallel and perpendicular to the galactic plane respectively.

In principle, the feature at $(80°, 0°)$ should be matched by another in the opposite tangent direction to the nearby ring caustic, at approximately $(-80°, 0°)$. Although there is a plausible feature there, it is much less compelling than the one in the $(+80°, 0°)$ direction. There are several reasons why it may not appear as strongly. One is that the $(+80°, 0°)$ feature is in the middle of the Sagittarius spiral arm, whose stellar activity enhances the local gas emissivity, whereas the $(-80°, 0°)$ feature is not so favorably located. Another is that the ring caustic in the $(+80°, 0°)$ direction has unusually small dimensions. This may make it more visible by increasing its contrast with the background. In the $(-80°, 0°)$ direction, the nearby ring caustic may have larger transverse dimensions.

4 The big flow

Our proximity to a ring means that the associated flows, i.e. those flows in which the caustic occurs, contribute very importantly to the local dark matter density. Using the results of refs. [3, 6, 8], and assuming axial symmetry of the caustic ring between us and the tangent point (approx. 1 kpc away from us), the densities and velocity vectors on Earth of the associated flows can be derived:

$$d^+ = 1.7\ 10^{-24}\ \frac{\text{gr}}{\text{cm}^3}\ ,\ d^- = 1.5\ 10^{-25}\ \frac{\text{gr}}{\text{cm}^3}\ ,\ \vec{v}^{\pm} = (470\ \hat{\phi} \pm\ 100\ \hat{r})\ \frac{\text{km}}{\text{s}}, \tag{6}$$

where $\hat{r}, \hat{\phi}$ and $\hat{z}$ are the local unit vectors in galactocentric cylindrical coordinates. $\hat{\phi}$ is in the direction of galactic rotation. The velocities are given in the (non-rotating) rest frame of the Galaxy. Because of an ambiguity, it is not presently possible to say whether $d^{\pm}$ are the densities of the flows with velocity $\vec{v}^{\pm}$ or $\vec{v}^{\mp}$. Eq. 6 has implications for dark matter searches. Previous estimates of the local dark matter density, based on isothermal halo profiles, range from 5 to 7.5 $10^{-25}\ \frac{\text{gr}}{\text{cm}^3}$. The present analysis implies that a single flow (d^+) has three times that much local density, i.e. that the total local density is four times higher than previously thought. The large size of d^+ is due to our proximity to a cusp of the nearby caustic. Assuming axial symmetry, that cusp is only 55 pc away from us. The exact size of d^+ is sensitive to our distance to the cusp but, in any case, d^+ is very large. If we are inside the tube of the fifth caustic, there are two additional flows on Earth, aside from those given in Eq. 6. A list of approximate local densities and velocity vectors for the $n \neq 5$ flows can be found in ref. [14]. An updated list is in preparation.

The sharpness of the rises in the rotation curve and of the triangular feature in the IRAS map implies an upper limit on the velocity dispersion δv_{DM} of the infalling dark matter. Caustic ring singularities are spread over a distance of order $\delta a \simeq \frac{R\ \delta v_{\text{DM}}}{v}$ where v is the velocity of the particles in the caustic, δv_{DM} is their velocity dispersion when they first fell in, and R is the turnaround radius then. The sharpness of the IRAS feature implies that its edges are spread over $\delta a \lesssim 20$pc. Assuming that the feature is due to the $n = 5$ ring caustic, $R \simeq 180$ kpc and $v \simeq 480$ km/s. Therefore $\delta v_{\text{DM}} \lesssim 53$ m/s.

There may be evidence for the accretion of baryonic matter onto the $n \leq 4$ rings as well. Binney and Dehnen studied [15] the outer rotation curve of the Milky Way and concluded that its anomalous behaviour can be explained if most of the tracers of the rotation are concentrated in a ring of radius 1.6 $r_\odot = 13.6$ kpc. This is very close to the expected radius (13.9 kpc) of the $n = 3$ ring. Recently, the SDSS collaboration detected [16] an overdensity of stars which appears to be lying in the galactic plane on an arc of circle at least $40°$ in length, and of galactocentric radius approximately 18 kpc. These stars have properties consistent with those of spheroid stars but their spatial distribution is not consistent with a power law spheroid. The observed feature may be due to the accretion of stars onto the $n = 2$ ring.

The caustic ring model may explain the puzzling persistence of galactic disk warps [17]. These may be due to outer caustic rings lying somewhat outside the galactic plane and attracting visible matter. Such disk warps would not damp and would persist on cosmological time scales.

The caustic ring model, and more specifically the prediction Eq. 6 of the locally dominant flow associated with the nearby ring, has important consequences for axion dark matter searches [18], the annual modulation [14, 19, 20] and signal anisotropy [21, 20] in WIMP searches, the search for γ-rays from dark matter annihilation [22], and the search for gravitational lensing by dark matter caustics [4]. The model allows precise predictions to be made in each of these approaches to the dark matter problem.

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Table 1: Radii at which rises in the Milky Way rotation curve start (r_1) and end (r_2), the corresponding increases in velocity Δv, the caustic ring radii a_n of the self-similar infall model in the two different fits (I and II) discussed in the text, and typical velocity increases $\bar{\Delta} v_n$ predicted by the model, in fit I, without amplification due to baryon accretion.

r_1 (kpc)	r_2 (kpc)	Δv (km/s)	a_n^{I} (kpc) n = 1 ..14	a_n^{II} (kpc) n = 1 .. 13	$\bar{\Delta} v_n$ (km/s) n = 1 .. 14
			41.2		26.5
			20.5	37.2	10.6
			13.9	18.6	6.8
			10.5	12.5	5.0
(8.28)	(8.38)	(12)	8.50	9.51	3.9
(7.30)	(7.42)	(8)	7.14	7.68	3.2
6.24	6.84	23	6.15	6.45	2.6
5.78	6.01	9	5.41	5.56	2.3
4.91	5.32	15	4.83	4.89	2.0
4.18	4.43	8	4.36	4.36	1.7
3.89	4.08	8	3.98	3.94	1.5
3.58	3.75	6	3.66	3.60	1.3
3.38	3.49	14	3.38	3.31	1.2
3.16	3.25	8	3.15	3.05	1.1